Comparison of two-color methods based on wavelength and adjacent pulse repetition interval length

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This paper describes the characteristics of a two-color method based on the adjacent pulse repetition interval length (APRIL), which functions as a length unit for femtosecond optical frequency combs (FOFCs), and compares the results to the wavelength-based two-color method. The wavelength-based two-color method can eliminate the inhomogeneous disturbance of effects caused by the phase refractive index; therefore, the APRIL-based two-color method can eliminate the air turbulence of errors induced by the group refractive index. Our numerical analysis of the APRIL-based two-color method will contribute to the pulse-laser-based two-color method, which secures traceability to the definition of the meter via APRIL.

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1 INTRODUCTION

The refractive index of air can significantly affect length measurements because the refractive index of air is a function of several air parameters such as temperature, pressure, humidity, and CO_2 concentration. For example, when the temperature increases by 1°C, the optical length decreases by approximately 1 ppm. Therefore, we need to correct lengths measured in air so that they may be treated as lengths measured in vacuum because only lengths measured in vacuum are comparable. One method for correcting the lengths measured in air requires an accurate measurement of the air parameters. When the air parameters are inhomogeneously distributed in the optical path, the averaged air parameters can be measured with several sensors [1]. However, the direct measurement of air parameters with high precision in an open-air field can be difficult. On the other hand, the inhomogeneous distribution of the refraction index can be corrected by a proposed method [2] for measuring the distance with two wavelengths (two colors) by utilizing the wavelength dependence of the refractive index of air. Several researchers [3]–[15] have studied this two-color method. For example, the correction of the phase refractive index of air over a path length of 61 m was reported to be better than 1.4×10^{-8} [15].

In 2009, the national standard of length in Japan was changed from the iodine-stabilized He-Ne laser to the femtosecond optical frequency comb (FOFC). Because the He-Ne laser is monochromatic light, its wavelength was used as a practical standard to realize the meter. On the other hand, FOFC is coherent polychromatic light. In addition to wavelength, FOFC contains another length unit, the adjacent pulse repetition interval length (APRIL), which is a coherent combination of multiple wavelengths. Distance measurements using APRIL are independently presented by Yamaoka et al. [16] and Ye [17] and are widely used by many other groups [18]–[21]. We have previously investigated length measurements that rely on APRIL instead of the wavelength [22]–[25], and we find that APRIL can be used as a low-cost measurement unit [25]. In this study, we examine the two-color method using APRIL and compare the properties of both wavelength-based and APRIL-based two-color methods using numerical simulations.

This paper is organized as follows. Section 2 briefly reviews the wavelength-based two-color method and its characteristics before describing the proposed method. The numeral experiments and their results are described in Section 3. Finally, our conclusions from this study are summarized in Section 4.

2 PRINCIPLES

2.1 Wavelength-based two-color method

For convenience, we summarize the basic concepts of the wavelength-based two-color method; the details of this method can be obtained elsewhere [10, 14, 26]. In a vacuum, the following relationship holds

$$\lambda_{\mathrm{vac}} = c_{\mathrm{vac}}/f, \tag{1}$$

where λ_{vac} is the wavelength, f is the frequency, and c_{vac} is the speed of light in vacuum. For the He-Ne laser, the frequency f is stabilized to the absorption line of an iodine cell. The wavelength λ_{vac} is used as a practical standard to realize the meter. The geometric distance L_{vac} is the true distance between two points (in vacuum). The distance measured by wavelengths

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in air is optical distance $L_{air}(\lambda_{vac})$. These distances have the following relationship:

$$L_{vac} = L_{air}(\lambda_{vac})/n_p(\lambda_{vac}, T, P, H), \tag{2}$$

$$\begin{aligned} L_{air}(\lambda_{vac}) &= (M_\lambda + N_\lambda) \times \lambda_{air} \\ &= (M_\lambda + N_\lambda) \times \lambda_{vac}/n_p(\lambda_{vac}, T, P, H), \end{aligned} \tag{3}$$

Here, $n_p(\lambda_{vac}, T, P, H)$ is the phase refractive index of the air, and T, P, and H are the temperature, barometric pressure, and humidity, respectively. The integer and fractional parts are M_λ and N_λ, respectively. The wavelength in air is λ_{air}. The phase refractive index of air is different for different wavelengths. The obtained optical distances $L_{air}(\lambda_{vac\_1})$ and $L_{air}(\lambda_{vac\_2})$, which are dependent on the respective wavelengths λ_{vac_1} and λ_{vac_2}, are different even though the wavelengths follow the same optical path. The geometric distance L_{vac_λ} can be obtained as follows [10, 14, 26]:

$$L_{vac\_\lambda} = L_{air}(\lambda_{vac\_2}) - A_\lambda \times [L_{air}(\lambda_{vac\_2}) - L_{air}(\lambda_{vac\_1})], \tag{4}$$

$$A_\lambda = \frac{n_p(\lambda_{vac\_2}, T, P, H) - 1}{n_p(\lambda_{vac\_2}, T, P, H) - n_p(\lambda_{vac\_1}, T, P, H)}. \tag{5}$$

Based on the previous studies [5, 6, 7, 10, 13, 14, 27, 28], the following characteristics for the wavelength-based two-color method are well known.

1. When the humidity is 0%, A_λ can be considered as a function of only two wavelengths. Using $L_{air}(\lambda_{vac\_1})$, $L_{air}(\lambda_{vac\_2})$, and A_λ, the parameter L_{vac_λ} can be determined with a precision on the order of 10^{-8}. Here, $L_{air}(\lambda_{vac\_1})$ and $L_{air}(\lambda_{vac\_2})$ are measurement values taken in an arbitrary environment, while A_λ is a value measured in the standard environment. (In this study, the standard environment has the following parameters: 20°C, 101.325 kPa, and 0% humidity.)

2. When the humidity is not 0%, A_λ changes with T and P. Using the value of A_λ measured in the standard environment, L_{vac} can be determined with a sufficiently good accuracy from $L_{air}(\lambda_{vac\_1})$ and $L_{air}(\lambda_{vac\_2})$.

2.2 APRIL-based two-color method

For convenience, we summarize the features of an FOFC; the details can be obtained elsewhere [29]. An FOFC is a stabilized pulse laser. In the frequency domain, a mode-locked laser generates equidistant frequency comb lines with the pulse repetition frequency f_{rep}. In the time domain, the electric-field packet repeats at the pulse repetition period $T_R = 1/f_{rep}$.

In a vacuum, the following relationship holds

$$\delta_{vac} = c_{vac}/f_{rep} \tag{6}$$

where δ_{vac} is APRIL, and f_{rep} is the pulse repetition frequency. Because f_{rep} is usually locked to the time standard with high precision (10^{-14} order), δ_{vac} can be used as a practical standard to realize the meter. In this way, the APRIL-based two-color method can be described similarly to the wavelength-based two-color method.

The distance measured by APRIL in air is called the optical distance $L_{air}(\lambda_{vac\_cen})$, where λ_{vac_cen} is the central wavelength of the pulse. The optical distance $L_{air}(\lambda_{vac\_cen})$ and the geometric distance L_{vac} have the following relationship:

$$L_{vac} = L_{air}(\lambda_{vac\_cen})/n_g(\lambda_{vac\_cen}, T, P, H), \tag{7}$$

$$\begin{aligned} L_{air}(\lambda_{vac\_cen}) &= (M_\delta + N_\delta) \times \delta_{air}(\lambda_{vac\_cen}) \\ &= (M_\delta + N_\delta) \times \delta_{vac}/n_g(\lambda_{vac\_cen}, T, P, H), \end{aligned} \tag{8}$$

where $n_g(\lambda_{vac\_cen}, T, P, H)$ is the group refractive index of the air, and M_δ and N_δ are the integer and fractional parts, respectively. The group refractive index n_g is defined by the following expression [30] based on the Edlén equations [31, 32]:

$$\begin{aligned} & n_g(\lambda_{vac\_cen}) \\ &\quad = n_p(\lambda_{vac\_cen}) - \lambda_{vac\_cen} \times \left[\frac{dn_p(\lambda_{vac})}{d\lambda_{vac}}\right]_{\lambda_{vac\_cen}} \end{aligned} \tag{9}$$

where $[dn_p(\lambda_{vac})/d\lambda_{vac}]_{\lambda_{vac\_cen}}$ is the derivative of the function $y = n_p(\lambda_{vac})$ at $\lambda_{vac} = \lambda_{vac\_cen}$. The phase refractive index n_p is calculated based on the equations given by Ref. [33]. The derivative of these equations calculates by numerical analysis software.

The group refractive index of air varies with respect to the central wavelength of the pulse. The optical distances $L_{air}(\lambda_{vec\_cen\_1})$ and $L_{air}(\lambda_{vec\_cen\_2})$, which are obtained by two pulses with the respective central wavelengths of $\lambda_{vec\_cen\_1}$ and $\lambda_{vec\_cen\_2}$, are different even though the two pulses with different central wavelengths follow the same optical path. The geometrical distance L_{vac_δ} can be obtained as follows:

$$\begin{aligned} L_{vac\_\delta} &= L_{air}(\lambda_{vac\_cen\_2}) \\ &\quad - A_\delta \times [L_{air}(\lambda_{vac\_cen\_2}) - L_{air}(\lambda_{vac\_cen\_1})], \end{aligned} \tag{10}$$

$$A_\delta = \frac{n_g(\lambda_{vac\_cen\_2}, T, P, H) - 1}{n_g(\lambda_{vac\_cen\_2}, T, P, H) - n_p(\lambda_{vac\_cen\_1}, T, P, H)}. \tag{11}$$

In an optical experiment, the optical distances $L_{air}(\lambda_{vac\_cen\_1})$ and $L_{air}(\lambda_{vac\_cen\_2})$ can be recorded by multiple pulse train interferences, which are proposed for recording the light reflected from different reflective surfaces [34]. We will verify the theory and characters of both two-color methods using simulation.

3 NUMERICAL SIMULATIONS

We used a polarization-mode-locked femtosecond fiber laser (Menlo Systems, FC1500) as a light source in this study. The central wavelength of the pulse was 1560.0 nm. The second wavelength used for both two-color methods was 780.0 nm, which was the second-harmonic generation of the FOFC source.

We first verify the situation when the humidity is not 0%. For example, we set humidity to 50%.

Figure 1 shows the change in the A factors of the refractive indices (namely, A_λ and A_δ based on Eqs. (5) and (11), respectively) for wavelengths 780.0 nm and 1560.0 nm as a function

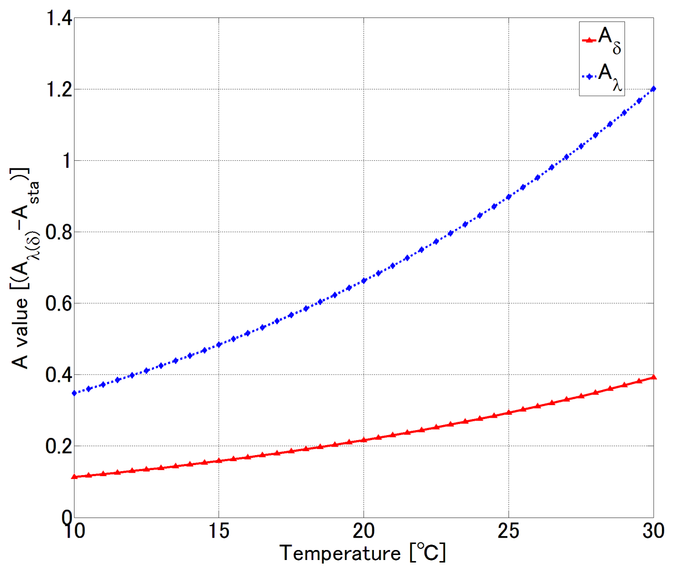

FIG. 1 Change in the A_λ and A_δ factors of the refractive indices with respect to changes in the temperature (humidity = 50%).

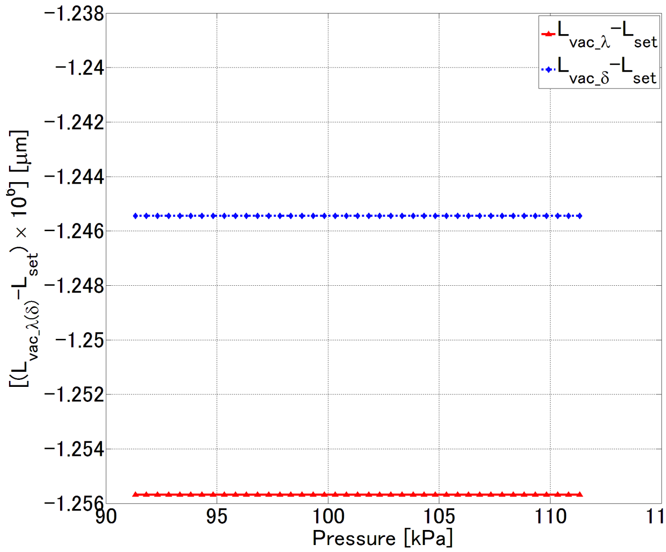

FIG. 4 Change in L_{vac_λ} and L_{vac_δ} with respect to changes in the barometric pressure (humidity = 50%).

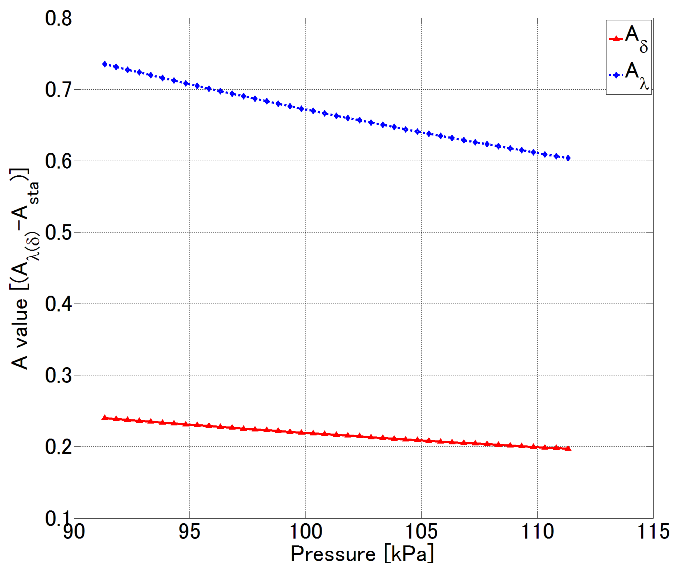

FIG. 2 Change in the A_λ and A_δ factors of the refractive indices with respect to changes in the barometric pressure (humidity = 50%).

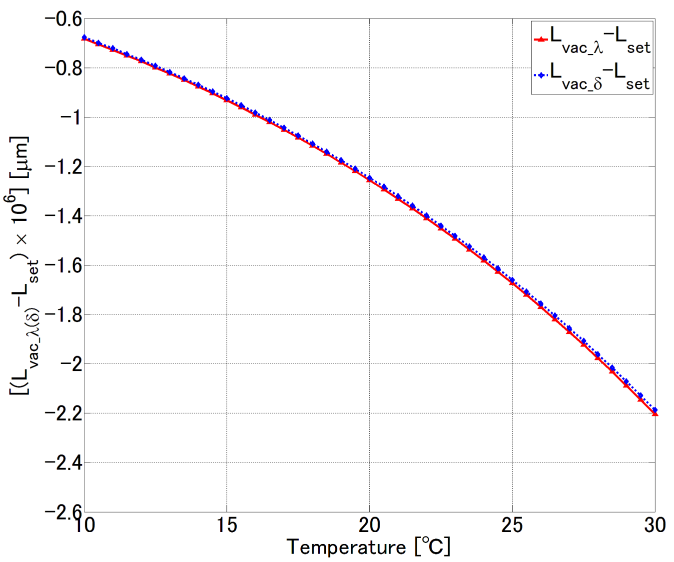

FIG. 3 Change in L_{vac_λ} and L_{vac_δ} with respect to changes in the temperature (humidity = 50%).

of the temperature in the 10°C to 30°C range (P = 101.325 kPa and H = 50% humidity). Figure 2 shows these changes as a function of barometric pressure changes between 91.325 kPa and 111.325 kPa (T = 20°C and H = 50% humidity). To clearly show the changes, we utilized an offset: A_{sta} for A_δ is -47.05296518 and A_{sta} for A_λ is -142.26466852. From Figures 1 and 2, we see that A_λ and A_δ were affected by changes in both the temperature and barometric pressure.

We now simulate length measurements using both two-color methods. First, L_{set} was arbitrarily set to a value on the order of several meters. Then, $L_{air}(\lambda_{vac\_1})$ and $L_{air}(\lambda_{vac\_2})$ were calculated using Eq. (2), and $L_{air}(\lambda_{vac\_cen\_1})$ and $L_{air}(\lambda_{vac\_cen\_2})$ were calculated using Eq. (7). The factors A_λ and A_δ were set to values from the standard state. We further calculated L_{vac_λ} and L_{vac_δ} on the basis of Eqs. (4) and (10), respectively. Figure 3 shows the changes of L_{vac_λ} and L_{vac_δ} from L_{set} as a function of the temperature change (P = 101.325 kPa and H = 50% humidity). The same calculation was performed for changes in barometric pressure (T = 20°C and H = 50% humidity), and Figure 4 shows the results. We see that L_{vac_λ}, and L_{vac_δ} were affected by temperature and barometric pressure changes.

Finally, we show the errors in L_{vec_λ} and L_{vec_δ} as a result of changes in humidity. The results are shown in Figures 5 and 6. From Figures 5 and 6, we understand the following: (1) from the estimate of the average air parameters, we will know if the value provided by the two-color method will meet the required measurement accuracy, and (2) in a laboratory measurement ($T \in [10,30]$°C, $P \in [80,120]$ kPa, and $H \in [10,80]$%), we can obtain a length measurement with an accuracy on the order of 10^{-6} with both two-color methods. To improve the accuracy of the two-color method, three-color methods [28, 35], and a method for measuring the factors A in a real environment [15] have been proposed.

We draw the following two conclusions from the computer simulation results: (1) there is no noticeable difference of disturbance elimination ability between the two-color meth-

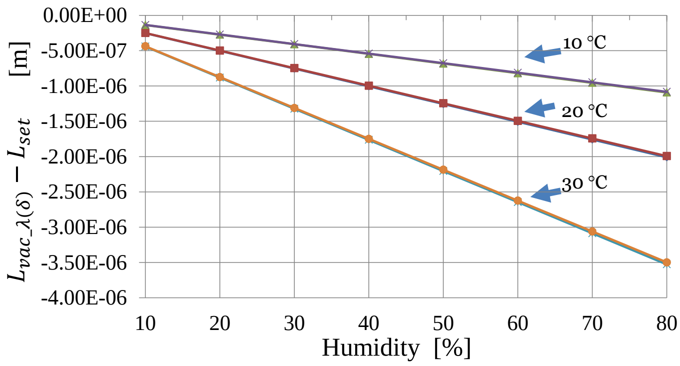

FIG. 5 Change in L_{vac_λ} and L_{vac_δ} with respect to changes in the temperature and humidity. The lines showing the difference between L_{vac_λ} and L_{vac_δ} overlap each other when T = 10°C, 20°C and 30°C, respectively.

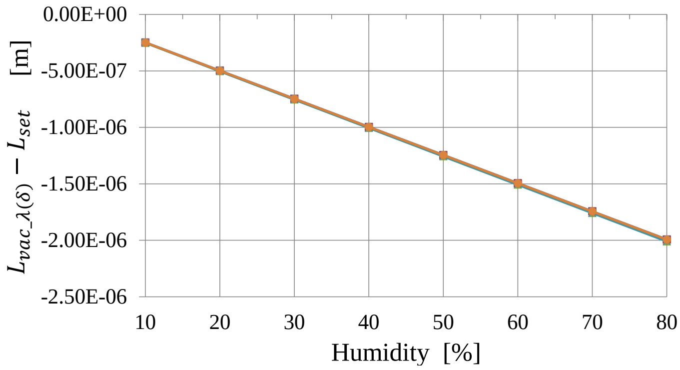

FIG. 6 Change in L_{vac_λ} and L_{vac_δ} with respect to changes in the barometric pressure and humidity. Note that all lines overlap when $P \in$ [80,120] kPa.

ods, and (2) because A_δ is smaller than A_λ, the APRIL-based two-color method theoretically has better accuracy than the wavelength-based two-color method. Note that the basic difference between the two methods is the object of elimination. The wavelength-based two-color method will only work for the phase refractive index. On the other hand, the APRIL-based method will only work for the group refractive index. Wavelength is the link between the traceability to the meter and the correction of the phase refractive index; then, APRIL is the link between the traceability to the meter and the correction of the group refractive index. Finally, we show the difference between the proposed method and the two-color method based on the time-of-flight method [3, 4, 8]. The proposed APRIL-based method secures traceability to the definition of the meter, whereas the time-of-flight method is based on recoding the time difference, which is not directly traceability to the meter.

4 CONCLUSION

We examined the wavelength-based and APRIL-based two-color methods. We conclude that APRIL can be used as a unit for the two-color method. Through a numerical simulation, we confirmed there was no noticeable difference between the two-color methods. In laboratory conditions, we can obtain a length measurement with an accuracy of 10^{-6} using either two-color method. Our results allow for the realization of a pulse-laser-based two-color method, which via APRIL (which is inversely proportional to pulse repetition frequency) secures traceability to the definition of the meter.

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